1-8 Properties of Exponents

What You'll Learn

1. To multiply or divide with exponents
2. To raise powers to powers
3. To use the rules for order of operations to simplify expressions

. . . And Why

To use scientific notation, and to solve algebraic problems involving exponents

PART 1 Multiplication and Division

Objective: Multiply or divide with exponents.

To multiply using exponential notation when we have the same base, we add the exponents. For example, $x^3 \cdot x^2 = x^{3+2} = x^5$. Let us consider a case in which one exponent is positive and one is negative.

$$b^5b^{-2} = b \cdot b \cdot b \cdot b \cdot b \cdot \frac{1}{b \cdot b} \quad \text{Using the definition of exponents}$$
$$= \frac{b \cdot b}{b \cdot b} \cdot b \cdot b \cdot b \quad \text{Using the associative property}$$
$$= 1 \cdot b \cdot b \cdot b$$
$$= b \cdot b \cdot b \quad \text{Using the identity property of multiplication}$$
$$= b^3$$

Notice that adding the exponents gives the correct result.

Theorem 1-8

For any nonzero real number a and integers m and n, $a^m a^n = a^{m+n}$.

(We can add exponents if the bases are the same.)

EXAMPLES Multiply and simplify.

1 $4^5 \cdot 4^{-3} = 4^{5+(-3)} = 4^2 = 16$ Adding exponents

2 $(-2)^{-3}(-2)^7 = (-2)^{-3+7} = (-2)^4 = 16$

3 $(8x^4y^{-2})(-3x^{-3}y) = 8 \cdot (-3) \cdot x^4 \cdot x^{-3} \cdot y^{-2} \cdot y^1$
$= -24(x^{4-3})(y^{-2+1})$
$= -24xy^{-1}$, or $\frac{-24x}{y}$

4 $(4x^a \cdot y^b)(2x^2y^3) = 4 \cdot 2(x^{a+2})(y^{b+3})$
$= 8(x^{a+2})(y^{b+3})$

Try This Multiply and simplify.

a. $8^{-3}8^7$

b. $(-3x^{-4})(25x^{-10})$

c. $(5x^{-3}y^4)(-2x^{-9}y^{-2})$

d. $(5x^my^n)(6x^7y^4)$

Theorem 1-9

For any real number $a \neq 0$ and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.

(We can subtract exponents if the bases are the same.)

EXAMPLES Divide and simplify.

5 $\frac{5^7}{5^{-3}} = 5^{7-(-3)}$ Subtracting exponents

$= 5^{7+3}$

$= 5^{10}$

6 $\frac{9^{-2}}{9^5} = 9^{-2-5}$

$= 9^{-7}$, or $\frac{1}{9^7}$

7 $\frac{7^{-4}}{7^{-5}} = 7^{-4-(-5)}$

$= 7^{-4+5} = 7^1 = 7$

8 $\frac{16x^4y^7}{-8x^3y^9} = \frac{16}{-8} \cdot \frac{x^4}{x^3} \cdot \frac{y^7}{y^9} = -2xy^{-2}$, or $-\frac{2x}{y^2}$

9 $\frac{14x^4y^7}{4x^5y^{-5}} = \frac{14}{4} \cdot \frac{x^4}{x^5} \cdot \frac{y^7}{y^{-5}} = \frac{7}{2}x^{-1}y^{12}$, or $\frac{7y^{12}}{2x}$

10 $\frac{18x^{5a}}{2x^{3a}} = \frac{18}{2} \cdot \frac{x^{5a}}{x^{3a}} = 9x^{5a-3a} = 9x^{2a}$

We do not define 0^0. Notice the following.

$$0^0 = 0^{1-1} = \frac{0^1}{0^1} = \frac{0}{0}$$

We have seen that $\frac{0}{0}$ is undefined, so 0^0 is also undefined.

Try This Divide and simplify.

e. $\frac{5^4}{5^{-2}}$ **f.** $\frac{10^{-2}}{10^{-8}}$ **g.** $\frac{42y^7x^6}{-21y^{-3}x^{10}}$

h. $\frac{33a^5b^{-2}}{22a^7b^{-4}}$ **i.** $\frac{56y^{ab}}{-7y^{ab}}$

Jo

Write a para
why "A who
tells how ma
a number by
definition of
exponent.

PART 2 Raising Powers to Powers

Objective: Use exponential notation in raising powers to powers.

Consider the expression $(5^2)^4$. It means $5^2 \cdot 5^2 \cdot 5^2 \cdot 5^2$, or 5^8. We can obtain the result by multiplying the exponents.

$$5^{2 \cdot 4} = 5^8$$

Consider $(8^{-2})^3$. It means $\frac{1}{8^2} \cdot \frac{1}{8^2} \cdot \frac{1}{8^2}$, or $\frac{1}{8^6}$, which is 8^{-6}.

Again, we could obtain the result by multiplying the exponents.

Theorem 1-10

For any nonzero real number a and integers m and n, $(a^m)^n = a^{m \cdot n}$.

(To raise a power to a power, we can multiply exponents.)

EXAMPLES Simplify.

11 $(3^5)^7 = 3^{5 \cdot 7} = 3^{35}$ Multiplying exponents

12 $(x^{-5})^4 = x^{-5 \cdot 4} = x^{-20}$, or $\frac{1}{x^{20}}$

Try This Simplify.

j. $(3^7)^6$ **k.** $(x^2)^{-7}$ **l.** $(t^{-3})^{-2}$

When there are several factors inside the parentheses, we can use the next theorem.

Theorem 1-11

For any nonzero real numbers a and b and integers m, n, and p, $(a^m b^n)^p = a^{m \cdot p} \cdot b^{n \cdot p}$

(To raise an expression with several factors to a power, raise each factor to the power by multiplying exponents.)

EXAMPLES Simplify.

13 $(3x^2y^{-2})^3 = 3^3(x^2)^3(y^{-2})^3 = 3^3x^6y^{-6} = 27x^6y^{-6}$, or $\frac{27x^6}{y^6}$

14 $(5x^3y^{-5}z^2)^4 = 5^4(x^3)^4(y^{-5})^4(z^2)^4 = 625x^{12}y^{-20}z^8$, or $\frac{625x^{12}z^8}{y^{20}}$

Try This Simplify.

m. $(2xy)^3$ **n.** $(-2x^4y^2)^5$ **o.** $(10x^{-4}y^7z^{-2})^3$

We now consider raising a quotient to a power. Consider $\left(\frac{5^5}{3^4}\right)^3$.

$$\left(\frac{5^5}{3^4}\right)^3 = \frac{5^5}{3^4} \cdot \frac{5^5}{3^4} \cdot \frac{5^5}{3^4} \cdot = \frac{5^{15}}{3^{12}}$$

Once more, we can obtain the result by multiplying the exponents. This is true in general, for positive, negative, or zero exponents.

Theorem 1-12

For any nonzero real numbers a and b and any integers m, n, and p, $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{m \cdot p}}{b^{n \cdot p}}$.

To raise a quotient to a power, raise both the numerator and denominator to the power by multiplying exponents.

EXAMPLES Simplify.

15 $\left(\frac{x^2}{y^{-3}}\right)^{-5} = \frac{x^{2(-5)}}{y^{-3(-5)}} = \frac{x^{-10}}{y^{15}} = x^{-10}y^{-15}$, or $\frac{1}{x^{10}y^{15}}$

16 $\left(\frac{2x^3y^{-2}}{3y^4}\right)^5 = \frac{(2x^3y^{-2})^5}{(3y^4)^5} = \frac{2^5x^{15}y^{-10}}{3^5y^{20}}$

$= \frac{2^5x^{15}}{3^5y^{30}} = \frac{32x^{15}}{243y^{30}}$, or $\frac{32}{243}x^{15}y^{-30}$

Try This Simplify.

p. $\left(\frac{x^{-3}}{y^4}\right)^{-3}$ q. $\left(\frac{3x^2y^{-3}}{2y^{-1}}\right)^2$

PART 3

Order of Operations

Objective: Use the rules for order of operations to simplify expressions.

When several operations, including raising to powers, are to be done in a calculation, we must decide in what order they are to be done. The agreements made about such calculations are given by the following rules.

Order of Operations

1. Calculate within innermost parentheses first.
2. Evaluate exponential expressions.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

EXAMPLES Simplify.

17 $3^2 - 9 \cdot 6$

$= 9 - 9 \cdot 6$ Evaluating the exponential expression first
$= 9 - 54$ Multiplying
$= -45$ Subtracting

18 $[2(8 - 13 + 2)^3 \div 6 + 2]^2$

$= [2(-3)^3 \div 6 + 2]^2$ Calculating within parentheses
$= [2(-27) \div 6 + 2]^2$ Simplifying the exponential expression
$= [-54 \div 6 + 2]^2$ Calculating within brackets
$= [-9 + 2]^2$ Dividing
$= [-7]^2$ Adding
$= 49$ Simplifying the exponent

Try This Simplify.

r. $3 \cdot 2^2 + 4$ **s.** $3 \cdot (2^2 + 4)$ **t.** $\{[(3 + 2)^2 - 3 + 2^2 + 1] \div 9\}^3$

Using Exponents

Scientific calculators have an exponential key y^x

Calculate 5^7.

5 y^x 7 = → 78125

Calculate 3^{-4}.

3 y^x 4 +/– = → 0.012345679

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

1-8 Exercises

A

Multiply and simplify.

1. $5^6 \cdot 5^3$ **2.** $6^2 \cdot 6^6$ **3.** $8^{-6} \cdot 8^2$ **4.** $9^{-5} \cdot 9^3$

5. $8^{-2} \cdot 8^{-4}$ **6.** $9^{-1} \cdot 9^{-6}$ **7.** $b^2 \cdot b^{-5}$ **8.** $a^4 \cdot a^{-3}$

9. $a^{-3} \cdot a^4 \cdot a^2$ **10.** $x^{-8} \cdot x^5 \cdot x^3$ **11.** $(2x^3)(3x^2)$ **12.** $(9y^2)(2y^3)$

13. $(14m^2n^3)(-2m^3n^2)$ **14.** $(6x^5y^{-2})(-3x^2y^3)$

15. $(-2x^{-3})(7x^{-8})$ **16.** $(6x^{-4}y^3)(-4x^{-8}y^{-2})$

17. $(5x^ay^b)(-6x^5y^9)$ **18.** $(-9x^my^6)(-8x^ny^p)$

Divide and simplify.

19. $\frac{6^8}{6^3}$ 20. $\frac{4^3}{4^{-2}}$ 21. $\frac{10^{-3}}{10^6}$ 22. $\frac{9^{-4}}{9^{-6}}$

23. $\frac{a^3}{a^{-2}}$ 24. $\frac{y^4}{y^{-5}}$ 25. $\frac{9a^2}{(-3a)^2}$ 26. $\frac{24a^5b^3}{-8a^4b}$

27. $\frac{-24x^6y^7}{18x^{-3}y^9}$ 28. $\frac{14a^4b^{-3}}{-8a^8b^{-5}}$ 29. $\frac{-18x^{-2}y^3}{-12x^{-5}y^5}$ 30. $\frac{-14a^{14}b^{-5}}{-18a^{-2}b^{-10}}$

31. $\frac{20x^{6a}}{-2x^a}$ 32. $\frac{-18x^{5y}}{-3x^{-6y}}$ 33. $\frac{36x^ay^b}{-12x^2y^5}$ 34. $\frac{-100x^{3a}y^{-5}}{-25x^{-a}y^6}$

Simplify.

35. $(4^3)^2$ 36. $(8^4)^{-3}$ 37. $(6^{-4})^{-3}$ 38. $(3x^2y^2)^3$

39. $(-2x^3y^{-4})^{-2}$ 40. $(-3a^2b^{-5})^{-3}$ 41. $(-6a^{-2}b^3c)^{-2}$ 42. $(-8x^{-4}y^5z^2)^{-4}$

43. $\left(\frac{4^{-3}}{3^4}\right)^3$ 44. $\left(\frac{5^2}{4^{-3}}\right)^{-3}$ 45. $\left(\frac{2x^3y^{-2}}{3y^{-3}}\right)^3$ 46. $\left(\frac{-4x^4y^{-2}}{5x^{-1}y^4}\right)^{-4}$

47. $3 \cdot 2 + 4 \cdot 2^2 - 6(3 - 1)$ 48. $3[(2 + 4 \cdot 2^2) - 6(3 - 1)]$

49. $4(8 - 6)^2 + 4 \cdot 3 - 2 \cdot 8 \div 4$ 50. $[4(8 - 6)^2 + 4] \cdot (3 - 2 \cdot 8) \div 4$

51. Find a counterexample to show that generally $(x^{-3})^{-3} \neq x^{-6}$. Is there a value of x for which this is true?

B

Simplify.

52. $\frac{(2^{-2})^{-4}(2^3)^{-2}}{(2^{-2})^2(2^5)^{-3}}$ 53. $\left[\frac{(-3x^2y^5)^{-3}}{(2x^4y^{-8})^{-2}}\right]^2$ 54. $\left[\left(\frac{a^{-2}}{b^7}\right)^{-3} \cdot \left(\frac{a^4}{b^{-3}}\right)^2\right]^{-1}$

55. $\left[\frac{(-4x^2y^3)(-2xy)^{-2}}{(4x^4y^2)(-2x^5y)}\right]^{-2}$ 56. $\frac{(3xy)^2(6x^2y^2) \times 4x^4y^4}{(4xy)^2 \times 13x^2y^2}$

57. ***Critical Thinking*** How can you use the definition of a negative exponent to make Theorem 1-12 a special case of Theorem 1-11?

Challenge

Simplify.

58. $(x^y \cdot x^{2y})^3$ 59. $(y^x \cdot y^{-x})^4$ 60. $(a^{b+x} \cdot a^{b-x})^3$

61. $(m^{a-b} \cdot m^{2b-a})^p$ 62. $(x^by^a \cdot x^ay^b)^c$ 63. $(m^{x-b}n^{x+b})^x(m^bn^{-b})^x$

64. $\left[\frac{(2x^ay^b)^3}{(-2x^ay^b)^2}\right]^2$ 65. $\left[\left(\frac{x^r}{y^s}\right)^2\left(\frac{x^{2r}}{y^{3s}}\right)^{-2}\right]^{-2}$

Mixed Review

Evaluate. 66. $t(3t + 5)$, for $t = 4$ 67. $-3x + 7 + 2x$, for $x = 5$ *1-3*

Simplify. 68. $\left(-\frac{1}{2}\right)\left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)\left(-\frac{4}{5}\right)$ 69. $(200)(-4)\left(-\frac{3}{2}\right)(0)(0.974)$ *1-2*

70. $3(x + 17) - 3(17 + x)$ 71. $7(14x - 15x) + 4(x + x)$ *1-3*

30. $\frac{7a^{16}b^5}{9}$ 41. $(-6)^{-2}a^4b^{-6}c^{-2}$ or $\frac{a^4}{36b^6c^2}$